

# **IWAM/Scattering of Internal Gravity Waves at Finite Topography**

Peter Muller  
University of Hawaii  
Department of Oceanography  
1000 Pope Road, MSB 429  
Honolulu, HI 96822  
phone: (808)956-8081 fax: (808)956-9164 email: [pmuller@hawaii.edu](mailto:pmuller@hawaii.edu)

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## **LONG-TERM GOAL**

The long term goal of the research project is the construction of a numerical model that

- predicts the internal wave field and internal wave induced transports regionally and globally, and
- can be used in conjunction with circulation, turbulence, acoustic and other models for research applications.

## **OBJECTIVES**

The internal wave model will be based on the integration of the radiation balance equation. The current project aims primarily at the

- formulation and calibration of the sink term that describes the dissipation of wave energy by wave breaking

A minor objective is the

- determination of the accuracy of the internal wave dispersion relation that neglects the meridional component of the earth's rotation.

## **APPROACH**

The radiation balance equation describes changes of the action density spectrum of the internal wave field along wave group trajectories caused by generation, transfer, and dissipation processes. The predicted quantity is the action density spectrum as a function of wavenumber, position, and time. The formulation of the dissipation sink term in the radiation balance equation is based on general theoretical and dimensional arguments. The calibration of free parameters will be done by comparison with observations and results from Large Eddy Simulation (LES) models.

Overall, the project will emulate the WAM project for surface gravity waves in its approach and methodology.

## WORK COMPLETED

1. Functional form of the dissipation sink term. Based on theoretical arguments by Hasselmann (1974) and Komen et al. (1994) a dissipation sink term of the form

$$S_{diss} = - c(\mathbf{k}) f(Ri^{-1}) E(\mathbf{k})$$

has been derived. Here  $\mathbf{k}$  is the wavenumber vector,  $E(\mathbf{k})$  the energy density spectrum,  $Ri$  the Richardson number, and  $c$  and  $f$  are two functions. The function  $c(\mathbf{k})$  determines which wavenumbers get dissipated by breaking events. We have chosen the functional form

$$c(\mathbf{k}) = c_0 (k / k_p)^q$$

with three adjustable parameters  $c_0$ ,  $k_p$  and  $q$ . This formula reproduces the formula of Garrett and Gilbert (1988) when  $q$  approaches infinity. The function  $f(Ri^{-1})$  determines the overall intensity of wave breaking and dissipation. We assume  $f$  to be a monotonic function of  $Ri^{-1}$ . The smaller the Richardson number the more vigorous is wave breaking.

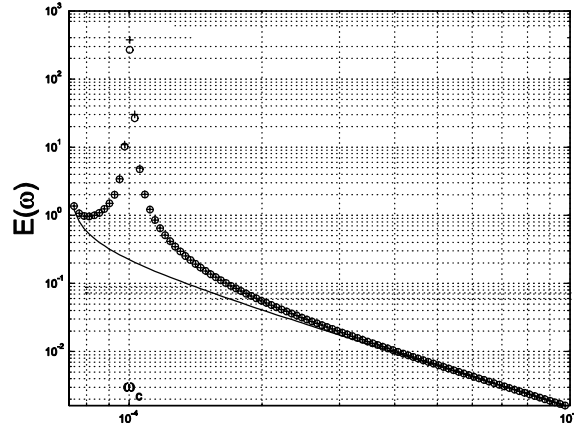
2. Solution of radiation balance equation for the reflection off a straight slope. To determine the free parameters  $c_0$ ,  $k_p$  and  $q$  and the functional form of  $f(Ri^{-1})$  the reflection of an incoming Garrett and Munk internal wave spectrum off a straight slope has been analyzed. Because of critical reflection the reflected spectrum has (infinitely) high shear and zero Richardson number with associated vigorous wave breaking. The dominant balance in the radiation balance equation is between propagation and dissipation

$$\mathbf{v}_n \cdot \partial E(\mathbf{k}) / \partial x_n = S_{diss}$$

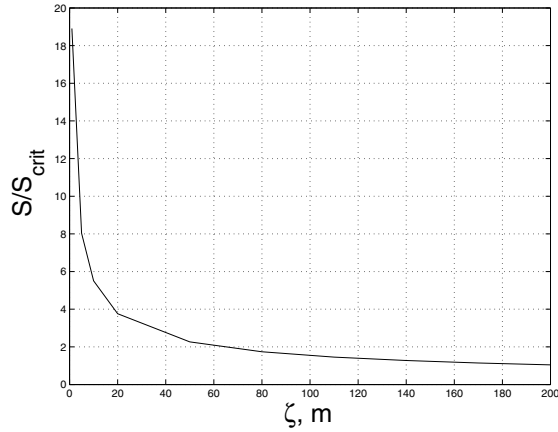
where  $\mathbf{v}$  is the group velocity and  $\mathbf{x}$  the position. The subscript  $n$  denotes the component normal to the slope. This reduced radiation balance equation has been solved for arbitrary monotonic  $f(Ri^{-1})$  and different values of the parameters  $c_0$ ,  $k_p$  and  $q$ . The solution is facilitated by the fact that the function  $f(Ri^{-1})$  can be eliminated from the problem by introducing a scaled distance  $\zeta$  instead of  $x_n$ . Spectra and their moments can then and have been calculated as a function of this rescaled distance  $\zeta$  for various values of  $q$ . The parameters  $c_0$  and  $k_p$  are absorbed into the function  $f$  during this elimination. As an example figure 1 shows the energy spectrum as a function of frequency for a parameter value  $q = 2$  and for two locations: right at the slope and at a distance  $\zeta_{crit}$  where the shear and Richardson number become non-critical. Also shown is the incident spectrum. The outgoing spectrum shows the characteristic peak at the critical frequency and very little decay in spectral amplitude except near the critical frequency. Figure 2 shows the decay of the total shear as a function  $\zeta$ , again for a parameter value of  $q=2$ . Comparison of this decay with the decay in physical space determines the function

$$f(Ri^{-1}).$$

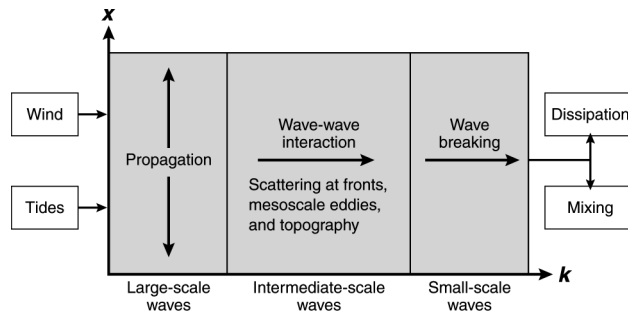
3. Effect of the meridional component of the earth's rotation. The "traditional approximation" neglects the local horizontal component of the earth's rotation. The dispersion relations with and without the meridional component have systematically been compared in mid-latitudes and at the equator. The difference and its dependence on frequency, wavenumber, latitude, and buoyancy frequency have been determined.



**Figure 1.** Energy density spectra  $2 E_i(\omega)$  [solid line],  $E_r(\omega) + E_i(\omega)$  at  $\zeta = 0$  (“+”) and at  $\zeta_{crit} \approx 200$  m (“o”) for  $q = 2$ . Environmental parameters are  $f = 7.3 \cdot 10^{-5} \text{ s}^{-1}$ ,  $N = 10^{-3} \text{ s}^{-1}$ ,  $\tan \theta_c = 0.07$  which corresponds to  $\omega_c \approx 10^{-4} \text{ s}^{-1}$



**Figure 2.** Total shear  $S(\zeta)$  relative to  $S_{crit} = 0.7 N^2$  as a function of the distance from the slope  $\zeta$ .



**Figure 3.** The dynamic balance of the oceanic internal wave field in physical ( $x$ ) and wavenumber ( $k$ ) space. The wind and tides generate large-scale waves of near inertial and tidal frequencies. These large-scale waves propagate away from their sources in physical space and cascade towards small-scale waves in wave-number space. The cascade is caused by wave-wave interactions and scattering at fronts, mesoscale eddies, topography, and other scatterers. The small-scale waves break and cause turbulence and mixing

## RESULTS

1. Radiation balance equation. The major dynamical processes that need to be included in the radiation balance equation are shown in Figure 3 (from Müller and Briscoe, 2000). These processes are:

- The generation of large scale waves at near-inertial and tidal frequency by the wind and the surface tide.
- The propagation in physical space of these large scale waves away from their sources at the surface and the bottom.
- The cascade of wave action in wavenumber space from large to small scales by wave-wave interaction and by scattering at fronts, eddies and topography.
- The breaking of small scale waves and conversion of their energy into mixing and heat.

A clear distinction must be made between near-inertial waves, internal tides and the internal wave continuum.

2. Dissipation function. The form of the dissipation function in the radiation balance equation needs to be specified before any meaningful simulations can be attempted. We have put forward a theoretically motivated form of this dissipation function which contains few free parameters and an arbitrary monotonic function. Our analysis of the reflection problem indicates that these parameters and function can be determined by comparing solutions with suitable observations or results from LES models. The most crucial pieces of information are:

- The total energy, shear or dissipation as a function of distance from the slope. This dependence will determine the function  $f(Rt^{-1})$ .
- The energy or shear spectrum as a function of wavenumber away from the slope. This dependence will determine the parameter  $q$ .

3. Neglection of the meridional component of the earth's rotation. Both the equatorial and mid-latitude analysis indicate that the error is largest for small buoyancy frequencies and low latitudes.

## IMPACT/APPLICATION

The development of a predictive dynamical model of the global or regional internal wave fields will have many benefits and applications.

Internal wave research will benefit from such a model since

- it will provide understanding of the internal wave field as a balance of generation, transfer and dissipation processes,

- it will focus research (it is expected that the proposed model will do for internal wave dynamics what the GM model did for internal wave kinematics), and
- it will predict changes of the internal wave field in response to changes in the forcing and environmental fields.

The dynamical internal wave model can be run in conjunction with circulation models, turbulence models, chemical tracer models, and biological population models where it would predict the internal wave induced transports, dispersion and mixing. In conjunction with acoustic transmission models the model would predict the internal wave induced “noise.”

## **TRANSITIONS**

## **RELATED PROJECTS**

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